Universal Physics Journal

Article IX: Galileo's Law of Constant Acceleration

Author: Ethan Skyler

Publication Date: September 9, 2002
Revision Date:

Purpose

Galileo Galilei had a fascination with falling objects. His extensive experimentation was undoubtedly spurred on by Aristotle's contention that large heavy objects fall faster than small light objects. Galileo reasoned that if one stone fell at a given rate of acceleration, and a second similar stone fell at the same rate of acceleration, for Aristotle to be right then after binding these two stones together, the resulting larger, heavier object should fall with a much greater rate of acceleration. This made no sense to Galileo for he could not think of a reason why the simple binding of these two stones should provide any cause for an increased rate of acceleration during a fall. Galileo had to wonder if Aristotle ever actually performed an experiment by dropping light and heavy stones together from a height to test his prediction. Galileo went on to perform his own experiments which revealed a truth contrary to Aristotle's thought experiment. Galileo was so encouraged by the power and truth of experiment that he spent years performing and writing about experiments involving the acceleration of falling objects. Over the course of these experiments, Galileo discovered some basic truths regarding the acceleration of objects. Having found these truths to be seldom published, it is my goal herein to once again bring to light the brilliance of Galileo's Law of Constant Acceleration.

Article IX

When Galileo set out to study the acceleration of a falling object, he was immediately beset with two problems. The first problem was the simple fact that the vertical fall of an object happened too quickly. In just a few seconds, the event was over for the falling object had impacted with the ground. The second problem was the timing of the increments of the object's fall. Galileo wondered if the falling object fell equal distances in equal units of elapsed time. But the precision timers we take for granted today did not exist back then. So Galileo had to resort to means of his own invention to first slow the rate of the object's fall and then to measure the passage of equal units of elapsed time during the event.

(2) To slow the pace of the event, Galileo selected a spherical metal ball which he allowed to freely roll down a long, straight groove cut into the surface of an inclined wooden plank. While introducing some curious effects due to the changing rates of rotation of the rolling object, this technique of Galileo's effectively reduced the object's rate of acceleration which made the accurate measurement of the event more achievable.

(3) To time the event, Galileo built a water timer where the water that dripped from a vessel at a constant frequency was first suddenly allowed to drip into a test pan at the start of the event and then suddenly prevented from dripping into the the pan at the event's end. After the event was

over, the water caught by the pan was very accurately weighed on a balance beam scale. In this manner, the amount of water in the test pan was a direct indication to Galileo as to the amount of time that had passed during the event. Although crude by today's standards, Galileo's water timer proved sufficiently accurate to enable his discovery of the Law of Constant Acceleration.

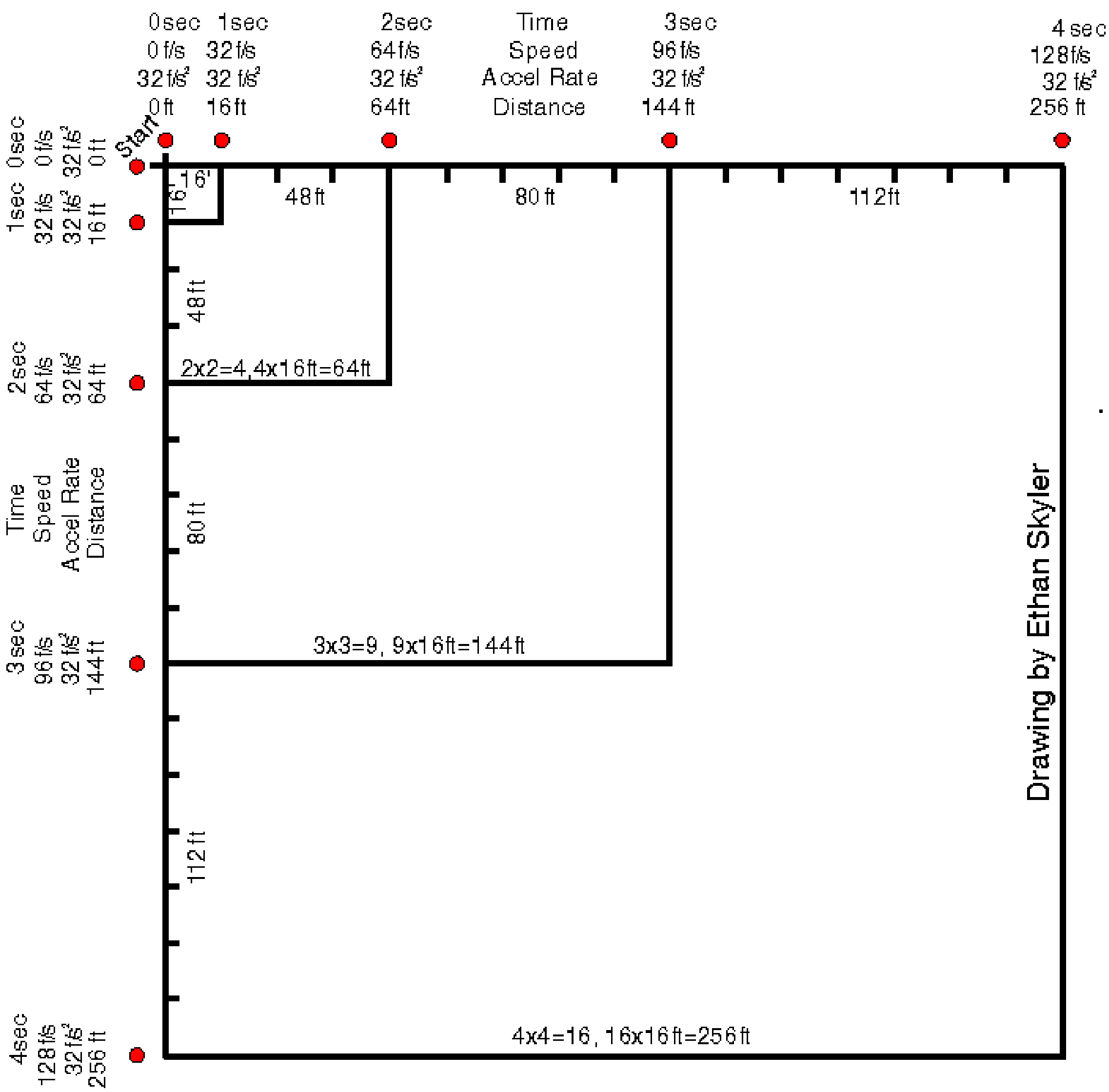
(4) Galileo soon discovered that a falling object did not traverse equal distances with the passage of equal units of time. Instead, his experiments revealed that the object's speed of fall increased with the passage of each measured unit of time. It was while analyzing the data from the falling object's rate of increase of speed (acceleration), that Galileo discovered the Law of Constant Acceleration. This law applies equally well to all examples of constant acceleration whether the object's acceleration is a vertical event caused by internal forces of gravitation toward Earth, the Moon, and the Sun, or instead is a horizontal event caused by external (contact) forces such as the momentarily constant acceleration experienced by an object inside an automobile or a rocket sled. As long as the object's rate of acceleration remains constant, Galileo's Law of Constant Acceleration is applicable.

(5) (Note: Galileo's law is sometimes referred to as the "Law of Fall". I see "fall" as a vague term in general use that gives not a hint as to whether acceleration is present or absent. For example, when a skydiver exits the basket of a high-altitude balloon, he or she suddenly experiences a weightless acceleration at the initial rate of 32 ft/s/s. This rate of acceleration, caused by the internal force of Earth gravitation being generated within each component of the skydiver's matter, immediately begins to diminish until, after a number of seconds of "fall", the skydiver's acceleration comes to a complete end. The skydiver has reached the uniform speed through the air known as "terminal velocity". For the remainder of this event, the skydiver will "fall" to Earth's surface in a uniform manner with acceleration generally absent.

Since Galileo's discoveries have to do with acceleration (a change in velocity including a change in speed or a change in direction) and even more specifically with constant acceleration in any direction, not just vertically down, and further since the vague non-scientific term "fall" makes no distinction between accelerated motion and uniform rest-motion, I think the "Law of Constant Acceleration" is an appropriate title for Galileo's discoveries.)

(6) While analyzing the data he collected from accelerating objects, Galileo discovered that the square of the time or duration of an object's constant rate of acceleration, multiplied by the unit of distance the object traveled during the first unit of time, yielded an accurate prediction of the total distance the object traveled from the point where its acceleration began. Understanding this discovery is not as complex as it may at first appear. Click on the icon to the right for a graphic representation of Galileo's "Times Square Law". First note how the constant acceleration of the red ball yields the same results in both a vertical event and a horizontal event. The base unit for distance is set during the first time-unit of acceleration. In this case, the time unit is 1 second. (The Times-Square predictions are equally accurate with the use of any other unit for time.) During the first second of constant acceleration, which in this event is at the rate of 32 ft/sec2, the red ball travels a distance unit of 16 feet. Understand that when an object is experiencing constant linear acceleration at the rate of 32 ft/sec2, at the end of each second of acceleration, the object's instantaneous velocity will be 32 ft/sec faster (or slower) than at the beginning of the

Galileo's Times-Square
Law of Constant Acceleration



second. In the red ball event, its relative velocity is 0 ft/sec at the start and 32 ft/sec at the end of the 1st second. The relative distance traveled by the red ball is found by averaging these two velocities. 0 ft/sec + 32 ft/sec = 32 ft/sec. 32 ft/sec / 2 = 16 ft/sec average velocity during the 1st second of the red ball's acceleration. With an average velocity of 16 ft/sec, the 1st second of travel will yield a Distance Unit of 16 feet. Now we are ready to apply Galileo's Times Square Law in predicting the relative distances of travel during the remaining seconds of the red ball's constant acceleration.

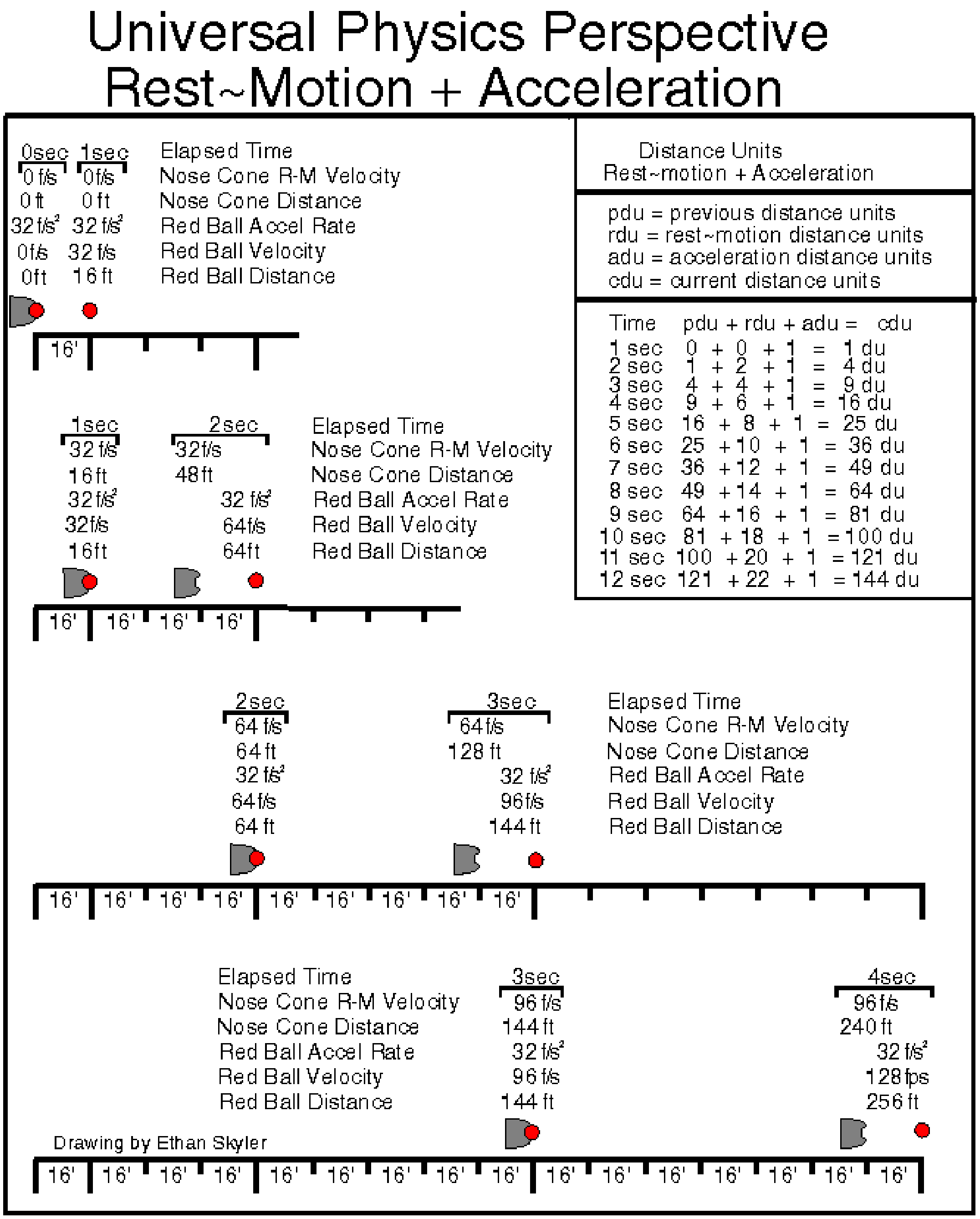
(7) As we proceed, keep in mind that in this event each Time Unit (tu) is 1 second while each Distance Unit (du) is 16 ft. As we perform the square of each Time Unit of acceleration, note that all squares share the same point of origin which is at the start or 0 sec. Although we already know the conditions of the first second of acceleration, realize that if a Time Unit of 1 is squared and the answer of 1 is multiplied times the Distance Unit of 1, the distance of travel for the red ball during its 1st second of acceleration is correctly predicted as 1 du or 16 ft.. Next, if the Time Unit of 2 is squared, the answer of 4 multiplied times the Distance Unit of 1 predicts a total distance of travel from the start of 4 du or 64 ft. Since each Times Square begins at the origin, as long as the Distance Unit is known, one is free to skip ahead to predict that after 5 Time Units of constant acceleration, the red ball will have traveled 52 x 1 du = 25 du or 400 ft from the origin. After 10 seconds of constant acceleration, the red ball will have traveled 102 x 1 du = 100du or 1600 ft from the origin. Discovery of this Times Square Law must have been quite exciting for Galileo. Here he was perhaps the first to witness an undeniable association between a Universal event involving constant acceleration and the precise predictions of a man-made mathematical process.

(8) Next Galileo turned his attention to the relationship between the predictions of the Times Square Law for each subsequent second. Second 1 predicts 1 distance unit (du), second 2 predicts 4 du, second 3 predicts 9 du , second 4 predicts 16 du and so on. From this information, Galileo discovered the "Odd-Number Law" when he noticed that the number 3 separates the predictions of seconds 1 & 2, the number 5 separates the predictions of seconds 2 & 3, the number 7 separates the predictions of seconds 3 & 4, and so on.... Thus he discovered that the advancing gap between his Times Square predictions was always filled by an advancing sequence of odd numbers. Overall, these two discoveries form Galileo's Law of Constant Acceleration.

(9) Galileo is best known for his light-hearted ability to question, dismantle and reassemble all of man's creations that, for him, lacked sensibility. Against his intellect and wit, no nonsensical concept could stand firm. If we learn anything at all from Galileo let it be this unusual ability of his. Question Everything - Accept Nothing. I say "unusual" for ironically, the polar opposite, "question nothing, accept everything" is the standard guide by which a student is most likely to succeed in ascending through the ranks of the modern version of the ancient science of Physics.

(10) Following Galileo's discoveries, in Galileo's tradition, have you formed any questions regarding the red ball event? I have a few. First of all, although I agree that the red ball is experiencing acceleration at a constant rate when friction is absent, which is due to the fact that in the vertical event the acceleration/Action force of gravitation of the red ball toward Earth remains generally constant during the ball's acceleration, I have a hard time resolving this recognition with the actual distances traveled. To me, if the red ball's acceleration is truly constant, then if it results in the ball traveling 16 ft in the 1st second, I can think of no reason why this same rate of acceleration should not continue to add 16 ft to the ball's travel during each subsequent second. Yet this prediction of mine appears to be unsupported by the chart above. On a separate issue, I am left in a complete state of wonderment as to exactly why it is that Galileo's Times Square Law so accurately predicts the outcome of constant linear acceleration. Overall I think that our understanding of constant acceleration will benefit if we spend some time applying the rest-motion based concepts of Universal Physics to this accelerational event.

(11) I will choose the horizontal event where the red ball is experiencing constant acceleration in a vacuum caused by a constant impressed external force of an unspecified nature. In the chart to the right, I have broken the ball's 4 seconds of acceleration into 4 separate one second events.



In each event I have inserted the rest-motion reference frame of what may be thought of as the nose cone of a non-accelerating rocket. My purpose here is to separate the distance units traveled by the ball's rest-motion from the distance units traveled by the ball's constant acceleration. This diminishment of an event into its base components is a technique often performed with great success by Galileo in his various analytical efforts.

(12) During the 1st second of the red ball's constant acceleration, the nose cone maintains its position at the start or point-of-origin with a speed of 0 ft/sec. I will refer to the relative distance from the point-of-origin the nose cone travels while in rest-motion as rest-motion distance units (rdu) which at this 1st one second interval is 0 rdu. Meanwhile the ball's acceleration causes it to advance 16 feet ahead of the "stationary" nose cone. I will refer to this 16 feet as the ball's acceleration distance unit (adu) which at this first interval is 1 adu. These are the base conditions for this event. From here I intend to show that during the entire event the red ball's adu never exceeds 1 during any second of its constant acceleration. If I am successful in this effort then I hope to finally understand how it is that the red ball's acceleration can remain constant while the distance it travels during each 1 second interval increases at a rate that is anything but constant.

(13) At the end of the 1st second, the ball's velocity is 32 ft/sec. If, for the moment, the a/A force accelerating the ball is suddenly brought to an end, in the next second, the ball will travel in the state of rest-motion for 2 distance units or 32 feet, relative to the start, with acceleration entirely absent. This role of traveling the rest-motion distance units (rdu) I will assign to the nose cone. Thus, with the a/A force reapplied only to the red ball, at the beginning of the 2nd second, the ball and nose cone are together at the 16 foot mark with the nose cone traveling in rest-motion at a steady 32 ft/sec relative to the start while the red ball is beginning its 2nd acceleration run ahead of the nose cone. During this 2nd second of time, the nose cone, with its velocity of 32 ft/sec, will travel 2 rdu or 32 feet while the red ball will begin the 2nd second with a velocity of 0 ft/sec relative to the nose cone and accelerate ahead to reach a velocity of 32 ft/sec relative to the nose cone. This means the ball's average velocity relative to the nose cone is 16 ft/sec which will again result in the red ball ending the 2nd second 16 ft or 1 adu ahead of the nose cone. When the nose cone's 2 rdu, relative to the start, are added to the red ball's 1 adu, relative to the nose cone, the combined total of 3 distance units is the result. Notice that just as in the 1st second event, here in the 2nd second event the red ball's acceleration accounts for just one of the distance units traveled. The remaining 2 du traveled by the nose cone would have been traveled had acceleration been entirely absent for the red ball at the beginning of the 2nd second. In other words, in both the 1st and 2nd seconds, the distance units advanced by the red ball remains constant at 16 ft or 1 distance unit..

(14) To set the scene for the 3rd second event, I will again increase the velocity of the nose cone to match the red ball's velocity at the end of the previous second which is now 64 fps. During the 3rd second event, the rest-motion velocity of the nose cone will cause it to travel 4 rdu or 64 ft relative to the start. Again the red ball will accelerate ahead of the nose cone causing it to travel 1 extra adu or 16 ft relative to the nose cone. A quick check of the 4th second event reveals that the red ball's acceleration again accounts for just 1 adu of travel. Therefore the earlier prediction that the red ball's acceleration, if constant, should account for the addition of a constant distance unit of travel for each second of acceleration turns out to be correct.

(15) Why then is this truth regarding constant acceleration not initially obvious? The acceleration of the red ball is clear to see during the 1st second event. It begins by accelerating away from a position with an observed rest-motion of 0 fps.. One second later the red ball is accelerating away from a position with an observed rest-motion of 32 fps. Then later still from a position with an observed rest-motion of 64 fps. Then 96 fps. Then 128 fps, and so on. All such speeds of rest-motion are determined by an observer who remains at rest relative to the 0 fps position at the start of the event. Yet the whole thing makes sense only when the observer is forced to adjust the speed of his or her rest-motion to match the rest-motion of the nose cone for each subsequent second of acceleration. This way the observer is always watching a 1st time unit event where the red ball is observed traveling 1 distance unit (16 feet in this event) while accelerating away from a position of rest (0 fps).

(16) This continual reassignment of a new velocity of rest-motion from which to observe the event of constant linear acceleration makes more sense when one considers the observation of constant centripetal acceleration. It is common practice to observe and calculate the constant centripetal acceleration of an object by the continual reassignment of a rest-motion path of observation set tangent to the object's circular path of motion. The observer's rest-motion path of observation is reassigned once for each time unit of the event with the observer's velocity set equal to the orbiting object's velocity at the instant of assignment. Then for the next time unit interval, any change in direction of the orbiting object represents the object's acceleration away from the tangential rest-motion path of observation. When the time unit interval is over, the observer's path is reassigned once again as tangent to the accelerating object's circular path of motion which means that for an instant the non-accelerating observer's velocity is a match with the accelerating object's velocity. This is not unlike my reassignment of the velocity of the non-accelerating nose cone to match for an instant with the velocity of the accelerating red ball at the beginning of the next time unit interval of the ball's constant acceleration. In plainer words, my solution of resetting a constant linear acceleration event at the beginning of each subsequent time unit interval is already accepted as common practice during events involving constant centripetal acceleration. This common practice extends at least as far back as Isaac Newton's time for he used just such a model as an aid in developing his form of the mathematical process of incremental change known as calculus.

(17) My next goal is to determine exactly why it is that Galileo's Times Square Law is correct regarding the distance units traveled by a constantly accelerating object. I will labor over this question for I find this direct relationship between the square of the time units of an object's constant acceleration and the number of distance units traveled by the accelerating object to be quite remarkable. Please consult the Distance Units Chart below. Notice Galileo's Times Square Law at work as the square of each time unit interval in the left-hand column is equal to the current distance units traveled from the start in the right-hand column.

(18) The information in the cdu column enabled Galileo to discover his "Odd-Number Law" regarding the advancing series of odd numbers separating the number of distance units traveled in subsequent intervals of time. In the Distance Units Chart above, the rest-motion distance units (rdu) are shown separately from the acceleration distance units (adu). In each time interval, the

distance traveled due to the red ball's acceleration is constant at 1 du or 16 feet. This is to be expected for the red ball's acceleration is constant in every interval. Thus the 1 du of travel caused exclusively by the ball's acceleration in the 1st interval can only be repeated without change in each subsequent interval.

(19) What is changing is the rest-motion distance units traveled by the nose cone. These rdu advance by 2 in each subsequent time unit interval. Why by 2? The key is the ball's acceleration rate which is 32 fps2. This rate dictates that when the nose cone's velocity of rest-motion is reset at the end of each time unit interval, it will change by 32 fps. Since our time unit is one second and our distance unit is 16 feet, this means that the nose cone's velocity has no other option but to change by 2 rdu per second from the rest-motion distance units traveled in the previous second. This 2 rdu change accounts for the odd-number increment that Galileo first observed.

(20) But what role does this 2 rdu change play in helping to shape Galileo's Times Square Law? This is the real question for which I am seeking an answer. Since I am working toward a solution as this is being written, at this point I do not know if I will eventually arrive at a satisfactory one. If not, I am prepared to leave this question unanswered for I think this is the best state to leave a question if no solution is clear and obvious to the author. This way when someone does develop the true solution, he or she will not have to wrestle needlessly with the author's supporters to displace a false solution before being granted the position of truth.

(21) I suspect the answer to my question will be revealed if we study the red ball event while keeping in mind all possible distance unit options available to the ball during its constant acceleration run. Beginning with the 1st second event, the square of 1 is 1 which predicts 1 du of travel. Since the value of the distance unit is set at the observed 16 feet of travel and verified by our knowledge that the red ball's average velocity for a 32 fps2 rate of acceleration is 16 fps causing it to travel the observed 16 feet during the 1st second of constant acceleration, I see here that there is no other option but to accept that 16 feet is one distance unit and further that 1 distance unit is the exact distance the red ball travels in the first time unit of one second. Thus the square of 1 correctly predicts 1 du of travel.

(22) During the 2nd second event, the square of 2 predicts 4 du of travel. Keep in mind that each square begins at the same point of origin or start. This means that the results of the previous square are included in the current square. Hence the 1 du of the 1st second event becomes the previous distance unit (pdu). For the predicted 4 du to be reached, 3 more du of travel must occur. Referring back to the rest-motion role of the nose cone in the 2nd second event, we know its velocity is a steady 32 fps as set by the red ball's acceleration rate of 32 fps2. Thus the nose cone has no option but to travel 32 feet or 2 distance units during the 2nd second event. Also set by the red ball's acceleration rate is the 16 ft or 1 adu of travel ahead of the nose cone. Here during the 2nd second event, the red ball has no other option. When these three components of travel are added, 1 pdu + 2 rdu + 1 adu, the current distance unit total will always equal 4 du just as predicted. Clearly, as long as the red ball's rate of acceleration remains constant, this can be the only outcome.

(23) During the 3rd second event, 5 more du of travel must occur. This is in keeping with Galileo's Odd-Number Law. Notice that the increase from 3 du to 5 du to 7 du is always 2 du. So each subsequent second event must add an additional 2 du over the previous event. Realize that 2 du each second in this event is 32 fps which is the exact increase in velocity offered by the red ball's rate of acceleration. This means the nose cone's velocity of rest-motion has no option but to increase 32 fps for each subsequent second. So the red ball's acceleration rate of 32 fps2 guarantees that the required 2 du of travel will be added to each subsequent second in order for the squaring of subsequent time units to predict the correct number of distance units of travel.

(24) While it can be argued that I have not presented the exact reason why Galileo's Times Square Law holds true, I think I have presented as fact the reason why no other option is possible. From the establishment of the length of 1 du, and the rate of increase of 2 du each second, the red ball's constant rate of acceleration governs all in a manner that makes the predictions of Galileo's Times Square Law come true.

Ethan Skyler

Author's Commentary

While presenting the brilliance of Galileo's Times Square Law discovery, the red ball event is also a good demonstration of the strength of the Universal Physics concept of rest-motion as portrayed by the reassignable role of the non-accelerating nose cone. Observations of each time unit made from the nose cone in the default state of rest-motion have provided us with a deeper understanding of the constancy of the red ball's rate of acceleration.

Copyright Notice